**Topics: Normal distribution, Functions of Random Variables**

1. The time required for servicing transmissions is normally distributed with *μ* = 45 minutes and *σ* = 8 minutes. The service manager plans to have work begin on the transmission of a customer’s car 10 minutes after the car is dropped off and the customer is told that the car will be ready within 1 hour from drop-off. What is the probability that the service manager cannot meet his commitment?
2. 0.3875
3. 0.2676
4. 0.5
5. 0.6987

ANS: B. 0.2676

1. The current age (in years) of 400 clerical employees at an insurance claims processing center is normally distributed with mean *μ* = 38 and Standard deviation *σ* =6. For each statement below, please specify True/False. If false, briefly explain why.
2. More employees at the processing center are older than 44 than between 38 and 44.
3. A training program for employees under the age of 30 at the center would be expected to attract about 36 employees.

ANS: A False. Because the probability for employees at the processing center are more between 38 and 44 than older than 44.

mean = 38 std = 6

age\_lessthan\_38 = stats.norm.cdf(38, loc = mean, scale = std)

age\_lessthan\_38 = 0.5

age\_less\_than\_44 = stats.norm.cdf(44, loc = mean, scale = std)

age\_less\_than\_44 = 0.841

age\_betweeen\_38\_and\_44 = age\_less\_than\_44 - age\_lessthan\_38

print('The probability of employee age between 38 and 44 is',np.round(age\_betweeen\_38\_and\_44,2),'%')

The probability of employee age between 38 and 44 is 34.13 %

age\_morethan\_44 = 1-stats.norm.cdf(44, loc = mean, scale = std1)

print('The probability of employee age more than 44 is',age\_morethan\_44,'%')

The probability of employee age more than 44 is 0.158 %

true\_or\_false = (age\_morethan\_44 > age\_betweeen\_38\_and\_44)

print('Answer:',true\_or\_false)

Answer: False

B. True. age\_36 = stats.norm.cdf(30, loc = mean, scale = std1)\*100

print('A training program for employees under the age of 30 at the center would be expected to attract about',np.round((age\_36\*400)/100,0),'employees')

A training program for employees under the age of 30 at the center would be expected to attract about 36.0 employees 30 = 0.0912 \* 400 = 36.48 (or 36 employees).

Therefore the statement B of the question is TRUE.

1. If *X1*~ *N*(μ, σ2) and *X*2 ~ *N*(μ, σ2) are *iid*normal random variables, then what is the difference between 2 *X*1 and *X*1 + *X*2? Discuss both their distributions and parameters.

ANS. The Normal Distribution has its link with the Central Limit Theorem, which states that ‘Any large sum of independent identically distribution random variables are approximately Normal then (X1 + X2) and (2X1) tends to have Normal distribution only If X1 and X2 are i.i.d and n is Large.

The Difference between 2X1 and (X1 + X2) is the magnitude they hold of two different sample subsets (X1 and X2) from the same source(population). X1 and X2 can be a different subset of a sample from a similar source (population) but If X1 ~ N(μ, σ2) then, 2 X1 ~ N(2 μ, 4 σ2 ) If X1 ~ N(μ, σ2) and X2 ~ N(μ, σ2) are iid normal random variables then (X1 + X2)(2 μ, 2 σ2) Hence, 2X1 – (X1+X2) ~(2 μ – 2 μ, 4 σ2 + 2σ2 ) The distribution remains the same for every sample subset of similar source, it tends to fall under Normal distribution and slight deviations in parameters.

The Normal distribution has two parameters, the mean, µ, and the variance, σ2. µ and σ2satisfy −∞ < µ < ∞, σ2> 0. We write X ∼ Normal (µ, σ2) or X ∼ N(µ, σ2 ).

1. Let X ~ N(100, 202). Find two values, *a* and*b*, symmetric about the mean, such that the probability of the random variable taking a value between them is 0.99.
2. 90.5, 105.9
3. 80.2, 119.8
4. 22, 78
5. 48.5, 151.5
6. 90.1, 109.9

ANS: D. 48.5,151.5

The probability of getting a value between a and b is 0.99.

So, the probability of getting value outside a and b is 1-0.99 = 0.01

The probability towards the left of a = -0.01/2 = -0.005

The probability towards the right of b = 0.01/2 = 0.005

Since we have probabilities of a and b, we need to calculate the probability of X, the random variable, at a and b, which has these probabilities.

To find the standard normal variable (z), we need to calculate X.

Z = (X - mue)/sigma

For a probability of 0.005, the Z value is -2.576.

For a:

a = mue+z(a)\*sigma

= 100+(-2.576)\*20

= 48.48

For b:

b = mue+z(b)\*sigma

= 100+2.576\*20

= 151.52

1. Consider a company that has two different divisions. The annual profits from the two divisions are independent and have distributions Profit1 ~ N(5, 32) and Profit2 ~ N(7, 42) respectively. Both the profits are in $ Million. Answer the following questions about the total profit of the company in Rupees. Assume that $1 = Rs. 45
2. Specify a Rupee range (centered on the mean) such that it contains 95% probability for the annual profit of the company.

ANS. Profit1 = ~ N(5, 32)

Profit2 = ~ N(7, 42)

Rate $1 = Rs. 45

Profit1 in Rupees = 5\*45 = 225 Rs.Million

Profit2 in Rupees = 7\*45 = 315 Rs.Million

Variance profit1 = (3\*45)^2 = 6075

Variance profit2 = (4\*45)^2 = 8100

Total Profit = 225+315 = 540

Total Variance = 6075 + 8100 = 14175

Standard Deviation = sqrt(14175) = 119.33

Z-score for 95% confidence = 1.96

540-1.96\*119.33 to 540+1.96\*119.33

= (305.77 Rs. Million to 774.23 Rs. Million).

1. Specify the 5th percentile of profit (in Rupees) for the company

ANS. Z-score for 5th percentile = -1.645

X = Total Profit +( Z-score \* Standard deviation)

= 540 + (-1.645 \* 119.33)

= 347.08 Rs. Million

1. Which of the two divisions has a larger probability of making a loss in a given year?

ANS. Probability of division 1 making a loss P(X<0)

stats.norm.cdf(0,5,3) = 0.04779

Probability of division 2 making a loss P(X<0)

stats.norm.cdf(0,7,4) = 0.04005